Warm-up Problems. Choose a few of these problems to work on as you get settled in today. You don’t need to complete all of the problems now. Once you’ve thought about a problem on your own, talk to someone sitting near you about your ideas.

1. The number 458 is written on a blackboard. It is allowed either to double the number on the blackboard, or to erase its last digit. How can we obtain the number 14 using these operations?

2. Which number is greater: $2^{300}$ or $3^{200}$? Why?

3. A three-digit number is given whose first and last digits differ by at least 2. We find the difference between this number and the reverse number (the number written with the same digits but in the reverse order). Then we add the result to its reverse number. What is the sum?

4. The numbers $p$ and $8p^2 + 1$ are both prime. Find $p$.

5. The numbers 1, 2, . . . , 20 are written on a blackboard. It is permitted to erase any two numbers $a$ and $b$ and write the new number $a + b - 1$. What number can be on the blackboard after 19 such operations?

6. The numbers 1, 2, . . . , 20 are written on a blackboard. It is permitted to erase any two numbers $a$ and $b$ and write the new number $ab + a + b$. What number can be on the blackboard after 19 such operations?

7. The next-to-last digit of the square of a natural number is odd. Prove that its last digit is 6.

8. Let $P$ be the parabola $y = x^2$, and let $C_1$ be the circle of radius 1 which is tangent to $P$ at two points. Iteratively define a sequence of circles $\{C_2, C_3, \ldots\}$, where each $C_n$ is located above and tangent at a point to $C_{n-1}$ and tangent to two points at $P$. Find the radius of $C_n$. 

![Diagram of parabola and circles]

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