A Panoply of Perplexing Expressions

The expressions below are arranged in no particular order. Choose one that looks interesting and decide how its value may be precisely defined as the limit of a sequence. Then try to discover the exact value by hand, checking your answer by computing a good numerical approximation. Finally, for the stout of heart, prove that your answer is correct. You are welcome to check any of your answers or proofs—just send them to me at <samv@math.stanford.edu>.

\[
\sqrt{2207 - \frac{1}{2207 - \frac{1}{2207 - \cdots}}}
\]

\[
\sqrt{\frac{2}{3}} + \sqrt{\frac{3}{4}} + \sqrt{\frac{4}{5}} + \cdots
\]

\[
\sqrt{\frac{1}{1 - \frac{1}{4} + \frac{1}{7} - \frac{1}{10} + \frac{1}{13} - \cdots}}
\]

\[
\sqrt{3\sqrt{9\sqrt{27\sqrt{81\cdots}}}}
\]

\[
\frac{1}{1} + \frac{1}{2} + \frac{2}{6} + \frac{3}{24} + \frac{5}{120} + \cdots + \frac{F_n}{n!} + \cdots
\]

\[
\sqrt{90 - \sqrt{90 - \sqrt{90 - \sqrt{90 - \cdots}}}}
\]

\[
\sqrt{\frac{3}{1 + \frac{2 + \frac{3}{1 + \frac{3}{1 + \cdots}}}{}^{}}}
\]

\[
\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + \cdots}}}}
\]