1 Line Drawing and Coordinate Transformation

A computer can draw line figures using only “move” and “draw” commands (which we’ll call \(M(x, y)\) and \(D(x, y)\), respectively). \(M(x, y)\) tells the computer to move the tip of an imaginary pen to the point \((x, y)\), but not to make a mark. \(D(x, y)\) tells the computer to draw a straight line from its current position to the point \((x, y)\). The following sequence of commands draws a square of side length 2 centered at the origin:

\[
M(-1, -1)D(1, -1)D(1, 1)D(-1, 1)D(-1, -1)
\]

The imaginary pen tip starts at \((-1, -1)\) and then is dragged to the three other square vertices and finally back to where it started.

**Exercise 1:** On the grid on the left below, pretend to be a computer and follow these instructions:

\[
M(0, 0)D(0, 6)D(4, 6)M(0, 3)D(3, 3).
\]

Next, work out a set of commands to draw the letter \(K\) on the grid to the right, below.

We can modify (transform) a set of instructions to obtain a new one. We’ll use notation like this: the transformation \(T(x, y) = (x + 2, y - 3)\) adds the number 2 to every \(x\)-coordinate and subtracts the number 3 from every \(y\)-coordinate. Or \(T(x, y) = (y, x)\) would swap the \(x\)- and \(y\)-coordinates.

For example, if we apply \(T(x, y) = (x + 2, y - 3)\) to the sequence 1 in Exercise 1 we obtain the following sequence:

\[
M(2, -3)D(2, 3)D(6, 3)M(2, 0)D(5, 0).
\]

**Exercise 2:** On the same grid that you plotted the original figure (left, above) plot the modified sequence generated by \(T(x, y) = (x + 2, y - 3)\). Do you see what happened and why it happened?
Exercise 3: On the following grids, start with the same instructions as previously:

\[ M(0, 0)D(0, 6)D(4, 6)M(0, 3)D(3, 3), \]

apply the following transformations to the sequence, and then draw the resulting sequence on one of the grids below. If you can, try to predict what the result will be before you do the calculations and drawing.

\[
\begin{align*}
  T(x, y) &= (1.5x, 1.5y) & T(x, y) &= (2x - 7, 2y - 5) \\
  T(x, y) &= (y, x) & T(x, y) &= (-y, x) \\
  T(x, y) &= (y + x, y - x) & T(x, y) &= (x + y, y)
\end{align*}
\]
Exercise 3: Using what you have learned with the previous examples, try to find the form of general transformations that do the following to your drawing. Use the grids below to test your ideas if you need them.

1. A transformation that moves the drawing by $t_x$ units in the $x$-direction and $t_y$ units in the $y$-direction.

2. A transformation that mirrors the drawing across the $x$-axis and a different transformation that mirrors it across the $y$-axis.

3. A transformation that rotates the figure by 90° clockwise and another that rotates a figure by 180°, both about the origin.

4. A transformation that magnifies the figure by a factor of $s$. What does this do if $s < 0$?

5. Can you find a transformation that rotates a figure by 90° clockwise about the point (3, 3)?