1. Consider two equilateral triangles $ABC$ and $ADE$ as in the picture (with $E$, $A$ and $B$ lying on one line in this order). Let $M$ be the midpoint of $B$ and $D$ and $N$ be the midpoint of $C$ and $E$ (see picture). Show that the triangle $AMN$ is also equilateral.

2. Let $ABC$ be an acute angled triangle. At each of the sides draw equilateral triangles which lie outside of the triangle $ABC$, i.e. choose points $D$, $E$ and $F$ such that $AFB$, $BDC$ and $CEA$ are equilateral triangles that intersect the triangle $ABC$ only in one edge each (see picture).

   a) Let $X$, $Y$ and $Z$ be the midpoints of the equilateral triangles $BDC$ and $CEA$ and $AFB$, respectively. Prove that the triangle $XYZ$ is equilateral.
   
   b) Prove that the three segments $AD$, $BE$ and $CF$ intersect in a common point.

3. Let $\omega$ and $\Gamma$ be two circles that are tangent to each other in point $P$ and such that $\omega$ lies inside $\Gamma$. Let $A$ and $B$ two points on $\Gamma$ such that the segment $AB$ is tangent to the circle $\omega$ in point $Q$. Assume that the line $PQ$ intersects the circle $\Gamma$ besides $P$ in a second point $C$.

   a) Prove that $C$ has equal distance from $A$ and $B$.
   
   b) Prove that $|CP| \cdot |CQ| = |CA|^2 = |CB|^2$ (here $|XY|$ denotes the distance of two points $X$ and $Y$).
4. Let $\omega_1$ and $\omega_2$ be two circles of different sizes that intersect each other in two points. Let $g$ and $h$ be the common tangents of $\omega_1$ and $\omega_2$ and let $P$ be the point where $g$ and $h$ intersect each other. Let $l$ be a line through $P$ and assume it intersects $\omega_1$ in the points $A$ and $C$ and $\omega_2$ in the points $B$ and $D$ and also assume that the points $A$, $B$, $C$, and $D$ lie in this order on the line $l$. Finally assume that $g$ is tangent to $\omega_1$ in $E$ and to $\omega_2$ in $F$.

a) Prove that $|PA| \cdot |PD| = |PB| \cdot |PC|$

b) Prove that $|PE| \cdot |PF| = |PA| \cdot |PD| = |PB| \cdot |PC|$.

5. Consider two equilateral triangles with equal side length and parallel sides, but one of them is pointing upwards and one pointing downwards (see picture). Suppose the two triangles intersect in a hexagonal region.

Let us now connect each vertex of this hexagon with the opposite vertex. Show that the resulting three segments intersect in a common point.

6. Let $ABCDEF$ be a hexagon. Assume that $AB$ is parallel to $DE$, $BC$ is parallel to $EF$, $CD$ is parallel to $FA$ and these three pairs of parallel lines all have the same distance (i.e. the distance between $AB$ and $DE$ equals the distance between $BC$ and $EF$ as well as the distance between $CD$ and $FA$). Furthermore assume that the angle $BAF$ is a right angle. Let $S$ be the intersection point of the segments $BE$ and $CF$. What is the size of the angle $BSC$ and why?

7. Consider a cyclic quadrilateral $ABCD$ (i.e. a quadrilateral where all four vertices lie on a common circle) and denote by $K$, $L$, $M$, $N$ the midpoints of the sides $AB$, $BC$, $CD$ and $DA$, respectively. Show that the orthocenters of the triangles $AKN$, $BLK$, $CML$ and $DNM$ form the vertices of a parallelogram.