In the following problems, a hat will be placed on heads of $n$ players. Each hat will have $k$ possible colors and there is unlimited supply of each colored hat. Players can see others hat colors but not their own. The rules of the game and winning condition are explained in each problem.

By playing the games with your teammates, try to come up with a strategy that maximizes your teams chance of winning. You need a lot of players in some problems, especially the last one.

(1) Two players, two colors. They will simultaneously guess their hat colors or pass. If at least one is right and no one guessed wrong, they win.

(2) Same game as above with three players and two colors.

(3) Two players, two colors. They will guess their own hat color simultaneously. If at least one guesses correctly, they win as a team. What is their best chance of winning?

(4) Same game as above with three players and three colors.

(5) Same game as above with seven players and seven colors.

(6) 100 prisoners in a line, 2 colors. This time, everyone sees hat colors of everyone in front. Starting from last person in line, they guess their hat colors. Those who guess right go free and the sentence is doubled for those who guess wrong. They want to minimize the number of prisoners.

(7) Same problem as above with 100 prisoners and 100 colors.

(8) Same problem as above with infinitely many prisoners (numbered as 1, 2, 3, . . . from last place) and two colors.