OM050: Advanced Problem Solving and Proof Techniques

Course Description
Advanced Problem Solving and Proof Techniques prepares students for subjects at the upper-division undergraduate level by developing their facility with abstract conceptual mathematics. Students gain experience analyzing complex problem situations, formulating solutions, rigorously justifying arguments, and presenting mathematical reasoning clearly and effectively, both orally and in writing. Course topics include general guidelines for analyzing problems, proving conditional and biconditional statements, the contrapositive method, working with negations, proof by contradiction, problem-solving heuristics, understanding quantifiers, mathematical induction, the construction method working with nested quantifiers, and special proof techniques. The course focuses on practical problem-solving and proof construction techniques that will be invaluable in many university level mathematics courses.

Learning Objectives
Upon completion of Advanced Problem Solving and Proof Techniques, students will demonstrate proficiency in:

- Applying techniques related to the above topics to solve novel problems
- Understanding the nature of rigorous mathematical reasoning
- Expressing mathematics clearly, in both written and oral communication
- Constructing valid mathematical arguments and distilling such arguments into complete and well-written mathematical proofs
- Using fundamental concepts related to sets, relations, ordering, functions, and counting theory as applied in a variety of mathematical subject areas

Required Textbook
*How to Read and Do Proofs: An Introduction to Mathematical Thought Processes*, Daniel Solow

Course Topics

- **Problem Solving Approaches**
  Phases of Problem Solving, Analyzing Problems, Generalization, Specialization, Utilizing Symmetry

- **Combinatorics**

- **Relations and Functions**
  Equivalence Relations, Partitions and Quotient Structures, Types of Orders, One-to-One and Onto Functions, Functions and Partitions, Indexed Families

- **Set Theory**
  Finite and Infinite Sets, Countable and Uncountable Sets, The Schröder-Bernstein Theorem, The Axiom of Choice and Well-orderings
- **General Proof Techniques**
  Logic and Proofs, Combining Forward and Backward Processes, Analyzing and Applying Definitions

- **Techniques from First-order Logic**
  Existential Quantifiers, The Construction Method, Universal Quantifiers, The Choose Method, Specialization, Nested Quantifiers, Quantifier Interchange

- **Techniques from Propositional Logic**
  Negations and Quantifiers, Counterexamples, Proof by Contradiction, The Contrapositive Method

- **Special Proof Techniques**
  Uniqueness Methods, Mathematical Induction, Well-ordering and Generalized Induction, Either/Or Methods, Max/Min Methods

**Overview of Assignments**

- **Participation:** For full participation credit, you will be required to present a solution to a problem in class at least 5 times during the semester.

- **Homework:** Weekly homework assignments will be posted.

- **Exams:** Four exams will be given during the semester, in addition to the formally proctored midterm exam and final exam.